

# MST124

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## TMA 02

## 2018J

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Covers Units 3, 4, 5 and 6

Cut-off date 23 January 2019

You can submit this TMA either by post to your tutor or electronically as a PDF file by using the University's online TMA/EMA service.

Before starting work on it, please read the document *Student guidance for preparing and submitting TMAs*, available from the 'Assessment tab' of the MST124 website.

The work that you submit should include your working as well as your final answers.

Your solutions should not involve the use of Maxima, except in those parts of questions where this is explicitly required or suggested. Your solutions should not involve the use of any other mathematical software.

Your work should be written in a good mathematical style, as described in Section 6 of Unit 1, and as demonstrated by the example and activity solutions in the study units. Five marks (referred to as good mathematical communication, or GMC, marks) on this TMA are allocated for how well you do this.

Your score out of 5 for GMC will be recorded against Question 10. You do not have to submit any work for Question 10.

If you have a disability that makes it difficult for you to attempt any of these questions, then please contact your Student Support Team or your tutor for advice.

**Question 1** – 5 marks

*You should be able to answer this question after studying Unit 3.*

The population of pine trees in a forest region is infected by a needle blight fungus that kills off trees, so that the number of trees decreases with time.

Suppose that  $n(t)$  is the number of pine trees in the region at time  $t$  (in years). Initially (at time  $t = 0$ ) there are 8000 trees. Assume that the number of pine trees is modelled by the exponential decay function

$$n(t) = 8000e^{kt} \quad (t \geq 0),$$

where  $k$  is a constant. After 3 years, 6000 trees remain.

- (a) Find the value of the constant  $k$ , correct to three significant figures. [3]
- (b) How long will it take for the number of trees to reduce to 3000? Give your answer in years to one decimal place. [2]

**Question 2** – 5 marks

*You should be able to answer this question after studying Unit 3.*

Use a table of signs to solve the inequality

$$\frac{4y + 1}{3 - 5y} \geq 0.$$

Give your answer in interval notation. [5]

**Question 3** – 16 marks

*You should be able to answer this question after studying Unit 3.*

- (a) This part of the question concerns the graph of the function

$$f(x) = x^2 - 4x + 3.$$

- (i) Complete the square to rearrange  $f(x)$  in the form

$$f(x) = (x - a)^2 - b,$$

where  $a$  and  $b$  are positive integers. [1]

- (ii) Hence explain how the graph of  $f$  can be obtained from the graph of  $y = x^2$  by using appropriate translations. [2]

- (iii) What is the image set of  $f$ ? [1]

*(You are not asked to sketch any graphs in this part, but you may find it helpful to do so.)*

- (b) This part of the question concerns the function

$$g(x) = x^2 - 4x + 3 \quad (2 \leq x \leq 4).$$

The function  $g$  has the same rule as the function  $f$  in part (a), but a smaller domain.

- (i) Sketch the graph of  $g$ , using equal scales on the axes. (You should draw this by hand, rather than using any software.) Mark the coordinates of the endpoints of the graph. What is the image set of  $g$ ? [3]

- (ii) Find the inverse function  $g^{-1}$ , specifying its rule, domain and image set. [6]

- (iii) Add a sketch of  $y = g^{-1}(x)$  to the graph that you produced in part (b)(i). Mark the coordinates of the endpoints of the graph of  $g^{-1}$ . [3]

**Question 4** – 10 marks

*You should be able to answer this question after studying Unit 4.*

- (a) Find all the solutions between  $-2\pi$  and  $2\pi$  of the equation

$$\cos \theta = -\frac{\sqrt{3}}{2},$$

giving all your answers as exact values in radians. [4]

- (b) In triangle  $ABC$  (with the usual notation), angle  $A = 29^\circ$ , side  $b = 11$  and side  $c = 8$ .

- (i) Use the cosine rule to find the length of side  $a$ , to two decimal places. [2]

- (ii) Without using the cosine rule again, find the remaining angles  $B$  and  $C$ , giving your answers to the nearest degree. [4]

**Question 5** – 10 marks

*You should be able to answer this question after studying Unit 4.*

- (a) Using the exact values for the sine and cosine of both  $\pi/4$  and  $\pi/6$ , and the angle difference identity for cosine, find the exact value of  $\cos(\pi/12)$ . [3]
- (b) Use the exact value of  $\cos(\pi/6)$  and the half-angle identity for sine to find the exact value of  $\sin(\pi/12)$ . [4]
- (c) Use the standard trigonometric identity  $\sin^2 \theta + \cos^2 \theta = 1$  to check the exact values you found in parts (a) and (b). [3]

**Question 6** – 10 marks

*You should be able to answer this question after studying Unit 5 and also Section 7 of the Computer Algebra Guide.*

Use Maxima to plot the parabola  $y = 4 - 3x - 2x^2$  and the circle  $x^2 + y^2 + 2x - 4y - 3 = 0$  on the same graph, and to find the coordinates of the points of intersection between the parabola and the circle.

State the values of the coordinates rounded to two decimal places, but do not attempt to use Maxima for rounding.

Include a printout or screenshot of your Maxima worksheet with your solutions. You are not expected to annotate your Maxima worksheet with explanations. [10]

**Question 7** – 16 marks

*You should be able to answer this question after studying Unit 5.*

The good ship Beegull is travelling in perfectly calm seas on course for a small remote island, to carry out a wildlife survey. It is travelling at a speed (relative to the surrounding sea) of  $40 \text{ km h}^{-1}$  with a bearing of  $55^\circ$ . It is initially on course to enter harbour at the island assuming no change in the sea conditions. Unfortunately, unknown to the crew, a strong current arises of  $15 \text{ km h}^{-1}$  in the direction from north-west to south-east.

Take unit vectors  $\mathbf{i}$  to point east and  $\mathbf{j}$  to point north.

- (a) Express the original velocity  $\mathbf{v}$  of the ship relative to the sea and the velocity  $\mathbf{c}$  of the current in component form, giving numerical values in  $\text{km h}^{-1}$  to one decimal place. [7]
- (b) Express the resultant velocity  $\mathbf{w}$  of the ship in component form, giving numerical values in  $\text{km h}^{-1}$  to one decimal place. [2]
- (c) Hence find the magnitude and direction of the resultant velocity  $\mathbf{w}$  of the ship, giving the magnitude in  $\text{km h}^{-1}$  to one decimal place and the direction as a bearing to the nearest degree. [5]
- (d) If the ship was 100 km from the island harbour when the current arose, what will be the shortest distance between the ship and the harbour when it misses the harbour because of the current? Give your answer to the nearest km. You may find it helpful to draw a diagram. You can assume there is no other land in the vicinity. [2]

**Question 8** – 18 marks

*You should be able to answer this question after studying Unit 6.*

This question concerns the function

$$f(x) = 2x^3 - 3x^2 - 12x + 20.$$

- (a) Find the stationary points of  $f$ , and give their exact  $x$ - and  $y$ -coordinates. [5]
- (b) Use the first derivative test to classify the stationary points that you found in part (a). [5]
- (c) Sketch the graph of  $f$ , indicating the  $y$ -intercept and the points that you found in part (a). (You should draw this by hand, rather than using any software, and you can use different scales on the two axes if appropriate.) [5]
- (d) Find the greatest and least values taken by  $f$  on the interval  $[-3, 3]$ . [3]

**Question 9** – 5 marks

*You should be able to answer this question after studying Unit 6.*

An object moves along a straight line. Its displacement  $s$  (in metres) from a reference point at time  $t$  (in seconds) is given by

$$s(t) = 2t^3 - 13t^2 + 8t \quad (t \geq 0).$$

- (a) Find expressions for the velocity  $v(t)$  and the acceleration  $a(t)$  of the object at time  $t$  and hence calculate the value of the velocity at time  $t = 2$ . [3]
- (b) Find any times at which the velocity of the object is zero. [2]

**Question 10** – 5 marks

A score out of 5 marks for good mathematical communication throughout TMA 02 will be recorded under Question 10. [5]

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